A NUMBER OF COMPUTER PROGRAMS FOR SCALING TECHNIQUES

by Jan G. Blom and Leo W.A. van Herpt

1. INTRODUCTION

Subjective judgements are, generally speaking, difficult to quantify. In speech research - especially in the evaluation of speech and voice appreciation - it is often the only possible technique available.

In order to meet an existing need a number of programs and techniques have been developed:

1. A scaling model for comparative judgements (Thurstone's model)
2. A scaling model for categorical judgements (Torgerson's model)
3. A multidimensional model for comparative judgements (Kruskal's model)
4. A technique for multidimensional scaling of categorical judgements.

A short description of the programs with a sample of three models follow below.

An application of the technique mentioned under 4. has been given in: "The Evaluation of Jury Judgements on Pronunciation Quality" in these Proceedings.

2. THURSTONE'S SCALING MODEL PROGRAM

2.1 General description.

The Thurstone scaling model program calculates scale values and the discriminal dispersions for an attribute of a number of objects from ordinal judgements concerning the relation between the objects. These judgements can be obtained for instance by means of the paired comparison technique with statements such as "A is larger than B", or by the ranking of a number of objects. The methods used in this program have been described by Torgerson (1967).

The computed scales are interval scales, and only the differences between the scale values are meaningful. All scale values may be increased or decreased by an identical number. The program calculates the scale values in such a way that the mean scale value becomes zero. The origin of the scale in itself has no meaning. If three objects on a scale possess the scale values of, say, zero, one and two it means that the difference between the third and the first object is twice the size of the difference between the second and the first object, not that the third object possesses twice as much of the measured attribute as the second object.

The program computes scale values and discriminal dispersions for the scaling models case IV and case V for complete and incomplete matrices.

The program computes a test value (Mosteller's test) for the goodness of fit of the scaling model.

2.2 Field of application.

The Thurstone scaling technique can be applied in a large number of areas of which we mention a few:

Judgement and 'measurement' of pronunciation
Judgement and 'measurement' of transmission quality
Psycho-acoustic investigation of e.g. pitch, loudness, duration, tonal purity, etc.
Investigation of prominence
Investigation as regards hierarchies of cues etc. etc.

2.3 The Thurstone scaling model.

The Thurstone scaling model presupposes that an attribute of a stimulus (object) acquires a certain value on a psychical continuum by a discriminal process. Thurstone defines this discrimination process as "that process by which the organism identifies, distinguishes or reacts to stimuli", without further defining the nature of the process. Each stimulus which is presented induces a discriminal process. Fluctuations in the organism induce fluctuations in the discriminal process, so that the value on the continuum is not always a constant one. When a stimulus is presented a great number of times, a frequency distribution of values is obtained on the continuum. Thurstone postulates that these frequency distributions follow a normal distribution. A stimulus is then defined by the mean value and the standard deviation of the values on the continuum with which the stimulus is associated. The mean value is defined as the scale value, the standard deviation as the "discriminal dispersion".

The information concerning the discriminal process and the value on the continuum cannot be obtained directly. This information has to be drawn from the judgements of the subjects concerning the relation between stimuli. These judgements may be given as follows: stimulus A is larger (louder, warmer, etc.) than stimulus B. The subjects are, as a matter of fact, supposed to be able to rank the stimuli on the basis of an attribute.

2.4 The THURS packet consists of 10 modules and 3 subroutines which are linked.

The partition into modules has been made in such a way that the core memory is used optimally.

The Thurstone scaling model program uses eight diskfiles.

File 1	contains the F'-matrix
File 2	contains the P'-matrix
File 3	contains the X'-matrix
File 4	contains the X''-matrix
File 5	contains the P''-matrix
File 6	contains the matrix with the number of subjects on which the P'-values have been based.
File 7	contains the M-matrix
File 8	is used for the re-ordening of matrices.

2.5 Program capacity.

The ppogram is suitabel for matrices with a 40 x 40 maximum. Therefore 40 objects can be scaled.

2.6 Minimal machine configuration.

CPU IBM 1131 with 8K core storage and one disk drive, 1442 card read punch, 1132 printer.

2.7 Test.

The program packet THURS was tested on IBM 1131 2B by monitor version 2, level 11.

2.8 Sample.

Output sample for case IV and V.

JOB 1 PAGE 1

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

NUMBER OF OBJECTS	8
NUMBER OF PAIRS	0
NUMBER OF SUBJECTS	0
INPUT MODE	2
THURSTONE CASE	4
PRINT F MATRIX	1
PRINT P MATRIX	1
PRINT X MATRIX	1
PRINT X* MATRIX	1
PRINT P* MATRIX	1

JOB 1 PAG 2

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 5

F MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.0	61.1	71.4	88.8	91.6	97.2	98.4	98.4
S8	37.8	0.0	60.1	75.3	82.3	91.3	95.3	96.2
S4	27.5	38.8	0.0	62.8	72.7	84.1	90.7	93.1
S6	10.1	23.6	36.1	0.0	68.5	95.1	98.9	96.6
S2	7.3	16.6	26.2	30.4	0.0	68.5	84.2	87.5
S5	1.7	7.6	14.8	3.8	30.4	0.0	86.0	83.0
S1	0.5	3.6	8.2	0.0	14.7	12.9	0.0	68.8
S3	0.5	2.7	5.8	2.3	11.4	15.9	30.1	0.0

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 5

P MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.00	0.61	0.72	0.89	0.92	0.98	0.99	0.99
S8	0.38	0.00	0.60	0.76	0.83	0.92	0.96	0.97
S4	0.27	0.39	0.00	0.63	0.73	0.85	0.91	0.94
S6	0.10	0.23	0.36	0.00	0.69	0.96	1.00	0.97
S2	0.07	0.16	0.26	0.30	0.00	0.69	0.85	0.88
S5	0.01	0.07	0.14	0.03	0.30	0.00	0.86	0.83
S1	0.00	0.03	0.08	0.00	0.14	0.13	0.00	0.69
S3	0.00	0.02	0.05	0.02	0.11	0.16	0.30	0.00

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 5

X MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.00	0.29	0.58	1.26	1.44	2.11	2.57	2.57
S8	-0.29	0.00	0.27	0.71	0.96	1.42	1.79	1.92
S4	-0.58	-0.27	0.00	0.34	0.62	1.03	1.38	1.56
S6	-1.26	-0.71	-0.34	0.00	0.50	1.76	0.00	1.99
S2	-1.44	-0.96	-0.62	-0.50	0.00	0.50	1.04	1.19
S5	-2.11	-1.42	-1.03	-1.76	-0.50	0.00	1.12	0.99
S1	-2.57	-1.79	-1.38	0.00	-1.04	-1.12	0.00	0.51
S3	-2.57	-1.92	-1.56	-1.99	-1.19	-0.99	-0.51	0.00

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 5

X* MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.00	0.50	0.84	0.92	1.45	1.95	2.47	2.70
S8	-0.50	0.00	0.33	0.41	0.94	1.44	1.96	2.19
S4	-0.84	-0.33	0.00	0.07	0.61	1.10	1.63	1.85
S6	-0.92	-0.41	-0.07	0.00	0.53	1.02	1.55	1.78
S2	-1.45	-0.94	-0.61	-0.53	0.00	0.49	1.02	1.24
S5	-1.95	-1.44	-1.10	-1.02	-0.49	0.00	0.52	0.75
S1	-2.47	-1.96	-1.63	-1.55	-1.02	-0.52	0.00	0.22
S3	-2.70	-2.19	-1.85	-1.78	-1.24	-0.75	-0.22	0.00

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 5

P* MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.49	0.69	0.80	0.82	0.92	0.97	0.99	0.99
S8	0.30	0.49	0.63	0.65	0.82	0.92	0.97	0.98
S4	0.19	0.36	0.49	0.53	0.72	0.86	0.94	0.96
S6	0.17	0.34	0.46	0.49	0.70	0.84	0.94	0.96
S2	0.07	0.17	0.27	0.29	0.49	0.68	0.84	0.89
S5	0.02	0.07	0.13	0.15	0.31	0.49	0.70	0.77
S1	0.00	0.02	0.05	0.05	0.15	0.29	0.49	0.58
S3	0.00	0.01	0.03	0.03	0.10	0.22	0.41	0.49

JOB PAG

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 5

OBJECT	SCALE VALUE	SIGMA
S7	-2.71	1.00
S8	-1.69	1.00
S4	-1.02	1.00
S6	-0.87	1.00
S2	0.19	1.00
S5	1.18	1.00
S1	2.24	1.00
S3	2.68	1.00

MOSTELLER'S TEST FOR GOODNESS OF FIT
CHI2 = 67.95 NDF = 20

JOB 1 PAG

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 4

F MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.0	61.1	71.4	88.8	91.6	97.2	98.4	98.4
S8	37.8	0.0	60.1	75.3	82.3	91.3	95.3	96.2
S4	27.5	38.8	0.0	62.8	72.7	84.1	90.7	93.1
S6	10.1	23.6	36.1	0.0	68.5	95.1	98.9	96.6
S2	7.3	16.6	26.2	30.4	0.0	68.5	84.2	87.5
S5	1.7	7.6	14.8	3.8	30.4	0.0	86.0	83.0
S1	0.5	3.6	8.2	0.0	14.7	12.9	0.0	68.8
S3	0.5	2.7	5.8	2.3	11.4	15.9	30.1	0.0

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 4

P MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.00	0.61	0.72	0.89	0.92	0.98	0.99	0.99
S8	0.38	0.00	0.60	0.76	0.83	0.92	0.96	0.97
S4	0.27	0.39	0.00	0.63	0.73	0.85	0.91	0.94
S6	0.10	0.23	0.36	0.00	0.69	0.96	1.00	0.97
S2	0.07	0.16	0.26	0.30	0.00	0.69	0.85	0.88
S5	0.01	0.07	0.14	0.03	0.30	0.00	0.86	0.83
S1	0.00	0.03	0.08	0.00	0.14	0.13	0.00	0.69
S3	0.00	0.02	0.05	0.02	0.11	0.16	0.30	0.00

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 4

X MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.00	0.29	0.58	1.26	1.44	2.11	2.57	2.57
S8	-0.29	0.00	0.27	0.71	0.96	1.42	1.79	1.92
S4	-0.58	-0.27	0.00	0.34	0.62	1.03	1.38	1.56
S6	-1.26	-0.71	-0.34	0.00	0.50	1.76	0.00	1.99
S2	-1.44	-0.96	-0.62	-0.50	0.00	0.50	1.04	1.19
S5	-2.11	-1.42	-1.03	-1.76	-0.50	0.00	1.12	0.99
S1	-2.57	-1.79	-1.38	0.00	-1.04	-1.12	0.00	0.51
S3	-2.57	-1.92	-1.56	-1.99	-1.19	-0.99	-0.51	0.00

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 4

X* MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.00	0.26	0.54	1.63	1.50	2.40	2.60	2.77
S8	-0.26	0.00	0.25	0.94	0.99	1.57	1.84	2.10
S4	-0.54	-0.25	0.00	0.52	0.66	1.13	1.43	1.73
S6	-1.63	-0.94	-0.52	0.00	0.40	1.17	1.67	2.08
S2	-1.50	-0.99	-0.66	-0.40	0.00	0.41	0.87	1.32
S5	-2.40	-1.57	-1.13	-1.17	-0.41	0.00	0.66	1.27
S1	-2.60	-1.84	-1.43	-1.67	-0.87	-0.66	0.00	0.65
S3	-2.77	-2.10	-1.73	-2.08	-1.32	-1.27	-0.65	0.00

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 4

P* MATRIX

	S7	S8	S4	S6	S2	S5	S1	S3
S7	0.49	0.60	0.70	0.94	0.93	0.99	0.99	0.99
S8	0.39	0.49	0.59	0.82	0.83	0.94	0.96	0.98
S4	0.29	0.40	0.49	0.70	0.74	0.87	0.92	0.95
S6	0.05	0.17	0.29	0.49	0.65	0.88	0.95	0.98
S2	0.06	0.16	0.25	0.34	0.49	0.66	0.80	0.90
S5	0.00	0.05	0.12	0.11	0.33	0.49	0.74	0.89
S1	0.00	0.03	0.07	0.04	0.19	0.25	0.49	0.74
S3	0.00	0.01	0.04	0.01	0.09	0.10	0.25	0.49

THURSTONE SAMPLE
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

THURSTONE SCALING MODEL CASE 4

OBJECT	SCALE VALUE	SIGMA
S7	-2.68	0.91
S8	-2.09	1.32
S4	-1.38	1.45
S6	-0.35	0.51
S2	0.27	1.05
S5	0.96	0.60
S1	1.95	0.87
S3	3.33	1.25

MOSTELLER'S TEST FOR GOODNESS OF FIT
CHI2 = 38.17 NDF = 13

3. SCALE MODEL PROGRAM PACKET FOR CATEGORICAL DATA

3.1 Introduction.

The scaling technique for categorical data computes scale values (M) and standard deviations (S) for objects which have been scaled on scales of successive categories. Furthermore, scale values (T) for the category boundaries are computed. The program computes a least-square solution for M, S and T by means of an iterative procedure. The terms of the sum of squares which is minimalized, are weighted with factors which are a function of the reliability of that particular observation. The computational process is stopped when the estimates for T have become stable. The use of the scaling technique and of the terminology can be found in Torgerson (1967).
The reader is referred to the article of Gertrude W. Diederich entitled: 'A general least squar⁻ solution for successive intervals', for the computing process .‹957).

3.2 Field of application.

The scaling technique for categorical data can be used for the 'measurement' of attributes of objects, which can be scored on a scale of successive intervals. These intervals need not be equally large.
The use of the scaling technique for categorical data results in a considerable saving of labour of the data-collecting when compared to the technique of paired comparisons (Thurstone's scaling model).

3.3 Description.

The program packet consists of four modules which are chained. The partition into the modules is such that the core storage capacity is used to an optimum.

3.4 Program capacity.

The program computes scale values, standard deviations and category boundaries for a maximum of 30 objects which are scored in maximally 10 categories.

3.5 Minimum machine configuration.

For the execution of the program packet it is necessary to have at one's disposal:

- IBM 1131 - model 2 B with one diskdrive
- 1442 Card Read Punch
- 1132 printer.

3.6 Test.

The program packet has been tested on an IBM 1130 configuration by Diskmonitor version 2, level 11.

3.7 Sample.

INPUT SAMPLE

```
// JOB

// XEQ TORG1     1

*FILES(1,T1),(2,T2),(3,T3),(4,T4),(5,T5),(6,T6),(7,T7),(8,T8)

1234     SAMPLE TORG

040403010303030303030300000300.000010

S1S2S3S4

DATA
```

```
                                     JOB     1234          PAGE    1

 SAMPLE TORG
 PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

 SCALING TECHNIQUE FOR CATEGORICAL DATA

 OPTIONS

 NUMBER OF OBJECTS                  4
 NUMBER OF CATEGORIES               4
 METHOD OF WEIGHTING                3
 INPUT MODE                         1
 PRINT F  MATRIX                    3
 PRINT FC MATRIX                    3
 PRINT P  MATRIX                    3
 PRINT X  MATRIX                    3
 PRINT W  MATRIX                    3
 PRINT X* MATRIX                    3
 PRINT P* MATRIX                    3
 PRINT S, M, T                      0
 MAX. NUMBER OF ITERATIONS         30
 CRITERIUM OF CONVERGENCE    0.000010
```

JOB 1234 PAGE 2

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

F MATRIX

1	S1	5000.0009	4772.0009	228.0000	0.0000
2	S2	3090.0004	3820.0004	2862.0004	228.0000
3	S3	228.0000	4772.0009	4987.0009	13.0000
4	S4	228.0000	1362.0002	5320.0009	3090.0004

JOB 1234 PAGE 3

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

FC MATRIX

1	S1	5000.0009 9772.001910000.001910000.0019
2	S2	3090.0004 6910.0009 9772.001910000.0019
3	S3	228.0000 5000.0009 9987.001910000.0019
4	S4	228.0000 1590.0002 6910.000910000.0019

JOB 1234 PAGE 4

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

P MATRIX

1	S1	0.5000	0.9772	1.0000
2	S2	0.3090	0.6910	0.9772
3	S3	0.0228	0.5000	0.9987
4	S4	0.0228	0.1590	0.6910

JOB 1234 PAGE 5

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

X MATRIX

1	S1	0.0000	1.9995	1.0000
2	S2	-0.4982	0.4982	1.9995
3	S3	-1.9995	0.0000	3.0117
4	S4	-1.9995	-0.9985	0.4982

JOB 1234 PAGE 6

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

W MATRIX

1	S1	1.0000	0.2059	0.0000
2	S2	0.9134	0.9134	0.2059
3	S3	0.2059	1.0000	0.0221
4	S4	0.2059	0.6897	0.9134

JOB 1234 PAGE 7

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

X* MATRIX

1	S1	0.0000	1.9994	5.0036
2	S2	-0.4987	0.4990	1.9981
3	S3	-2.0003	0.0002	3.0062
4	S4	-1.9970	-0.9997	0.4986

JOB 1234 PAGE 8

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

P* MATRIX

1	S1	0.5000	0.9772	0.9999
2	S2	0.3089	0.6911	0.9771
3	S3	0.0227	0.5001	0.9986
4	S4	0.0229	0.1587	0.6909

JOB 1234 PAGE 9

SAMPLE TORG
PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

SCALING TECHNIQUE FOR CATEGORICAL DATA

NR	L	SCALE VALUE	SD
1	S1	-1.0277	0.5693
	T1	-1.0277	
2	S2	-0.4586	1.1409
3	S3	0.1105	0.5690
	T2	0.1106	
4	S4	1.2518	1.1414
	T3	1.8210	

NUMBER OF ITERATIONS 13

4. KRUSKAL SCALING MODEL PROGRAM PACKET

4.1 Introduction.

The Kruskal scaling model estimates the coordinates of a configuration of n points situated in a multi-dimensional space with Minkowski's r-metric, (Euclidean metric and City-block metric are special cases of this metric), so that the distances between the points are a monotonous function of the measure of dissimilarity of n objects. In general the monotony demand cannot be fully met.
The degree to which there is a deviation from monotony is expressed in a measure for 'goodness of fit', called stress. The essence of the scaling technique is the determination of the configuration with minimal stress.
A number of problems arise:

1. How many dimensions should be chosen?
2. Which metric should be chosen?
3. Has a local minimum been found or an absolute one?

The answer to points one and two is usually determined by the demands made on the model and the theoretical assumptions of the investigator. An answer to the third point is found by trial and error. The carrying out of a number of computations with varying starting configurations give an insight into the nature of the minima found. If no solution with low stress is found the estimates of the dissimilarity may have been unreliable, so that the true ranking order is disturbed.
In a number of cases some notion of this disturbance can be obtained by carrying out a test of concordance between the subjects (W-test).
The publications of J.B. Kruskal (1964) give a full reference of the use of this scaling technique, the terminology and the many options possible.

4.2 Field of application.

The Kruskal scaling model can be used for the study of contrasts or of relations between objects (such as vowel signals, positions of mouth, etc.) which cannot be described with a linear attribute.

4.3 Description of program.

The program packet consists of fourteen modules which can be chained.
The partition into modules is such that the core storage capacity is optimally used.

4.4 Program capacity.

The input of (dis)similarities from card or disk or the input from matrices can be done to a capacity of maximally thirty objects ($30 \times 29/_2 = 435$ (dis)similarities) in a symmetrical experiment.

The input of raw data from a completely triadic experiment can be done to the capacity of maximally sixteen (16) objects ($16 \times 15 \times 14/_6 = 560$ triads). See 4.7.

Core storage is reserved for a maximum of thirty (30) points in a space of maximally five (5) dimensions.

4.5 Minimum machine configuration.

For the executions of the program packet one should have at one's disposal:

- IBM 1131 - model 2B with one diskdrive
- 1442 Card Read Punch
- 1132 printer
- 1627 plotter.

4.6 Test.

The program packet has been tested on an IBM 1130 configuration by Disk monitor version 2, level 11.

4.7 Manner of scoring for a completely triadic experiment.

In a triadic experiment the subjects are offered objects in groups of three, e.g. the objects I, J and K. The subjects are asked to indicate which two objects of the three given are most alike and then which couple is most dissimilar.

Suppose I and J to be the objects with the greatest similarity, then a one (1) is added in a cell (I, J) of a scoring matrix. Suppose again that I and K are the objects with the greatest dissimilarity, then a one (1) is subtracted in a cell (I, K) of the scoring matrix.

This process is continued for all combinations of three objects and for all subjects. The sum of cell (I, J) and cell (J, I) is the similarity score of I and J.

Incorrect scores of subjects, e.g. a score which states that I and J have both the greatest similarity and the greatest dissimilarity do not influence the similarity score (they only increase the error of the ranking order).

Missing scores of the subjects are omitted.

The score of one subject is given by two numbers in a triad. The first number indicates the ranking number of the object which does not belong to the pair with the greatest similarity, the second number indicates the ranking number of the object which does not belong to the pair with the greatest dissimilarity.

4.8 SAMPLE

The following sample (JOB 3456) is constructed from synthetical data.
The real distance between all pairs from 15 points in a 3-dimensional space with Euclidean metric is used as dissimilarity-measure.

```
SAMPLE 15 PUNTEN IN R3. ERRORLESS DATA.                         JOB   3456  PAGE 1
1130-PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE

NUMBER OF OBJECTS                 15
NUMBER OF DIMENSIONS               3
NORM OF SPACE                   2.00
CONFIGURATION OPTION               1
NUMBER OF (DIS)SIMILARITIES      105
(DIS)SIMILARITY OPTION             2
TIE OPTION                         1
INPUT MODE                         2
CORRELATION OPTION                 1
INITIAL STEP-SIZE               0.20
LOCAL MINIMUM CRITERION       0.0500
MAX NUMBER OF ITERATIONS         200
ROTATION OPTION                    2
ORIENTATION OPTION                11
OUTPUT DISTANCES                   3
OUTPUT (DIS)SIMILARITY MATRIX      3
OUTPUT DISTANCE MATRIX             3
OUTPUT CONFIGURATION               3
PLOT PROJECTIONS                   2
```

SAMPLE 15 PUNTEN IN R3, ERRORLESS DATA. JOB 3456 PAGE 4
1130-PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE

NR	T		P1-P2	(DIS)SIM	DIST	DIST'
81	0	1- 3	01-03	0.17200E 01	0.17062E 01	0.17062E 01
82	0	3-15	03-15	0.17651E 01	0.17639E 01	0.17639E 01
83	0	2-11	02-11	0.17802E 01	0.17639E 01	0.17639E 01
84	0	4-12	04-12	0.18149E 01	0.18103E 01	0.18103E 01
85	0	1-11	01-11	0.18239E 01	0.18156E 01	0.18156E 01
86	0	5-14	05-14	0.18355E 01	0.18304E 01	0.18304E 01
87	0	7-14	07-14	0.18545E 01	0.18304E 01	0.18304E 01
88	0	2-10	02-10	0.19014E 01	0.18990E 01	0.18990E 01
89	0	2- 7	02-07	0.19046E 01	0.18990E 01	0.18990E 01
90	0	2-15	02-15	0.19766E 01	0.20106E 01	0.20106E 01
91	0	10-12	10-12	0.20812E 01	0.20856E 01	0.20856E 01
92	0	4- 7	04-07	0.20861E 01	0.20931E 01	0.20931E 01
93	0	2- 9	02-09	0.21501E 01	0.21387E 01	0.21387E 01
94	0	2- 3	02-03	0.21644E 01	0.21387E 01	0.21387E 01
95	0	3-12	03-12	0.21698E 01	0.21539E 01	0.21539E 01
96	0	3-14	03-14	0.21751E 01	0.21576E 01	0.21576E 01
97	0	7-15	07-15	0.22015E 01	0.21966E 01	0.21966E 01
98	0	11-14	11-14	0.22067E 01	0.21966E 01	0.21966E 01
99	0	11-12	11-12	0.22108E 01	0.22031E 01	0.22031E 01
100	0	7-11	07-11	0.22249E 01	0.22310E 01	0.22310E 01
101	0	9-12	09-12	0.22280E 01	0.22310E 01	0.22310E 01
102	0	2-14	02-14	0.22346E 01	0.22566E 01	0.22566E 01
103	0	12-14	12-14	0.22417E 01	0.22640E 01	0.22640E 01
104	0	4-14	04-14	0.22699E 01	0.22769E 01	0.22769E 01
105	0	12-15	12-15	0.23387E 01	0.23681E 01	0.23681E 01

STRESS= 0.1039248E-06 AFTER 200 ITERATIONS

Pages 2 and 3
containing same
sort of information
are omitted.

1130-PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE

MATRIX OF (DIS)SIMILARITIES

	01	02	03	04	05	06	07	08
01	***********	1.14953	1.72004	1.63498	0.96237	0.33867	1.20938	0.46892
02	1.14953	***********	2.16449	1.35723	0.85697	1.27887	1.90466	1.43191
03	1.72004	2.16449	***********	1.61837	1.45825	1.63344	1.08853	1.58988
04	1.63498	1.35723	1.61837	***********	0.71317	1.47309	2.08615	1.51224
05	0.96237	0.85697	1.45825	0.71317	***********	0.88597	1.54081	0.97236
06	0.33867	1.27887	1.63344	1.47309	0.88597	***********	1.32393	0.16222
07	1.20938	1.90466	1.08853	2.08615	1.54081	1.32393	***********	1.32382
08	0.46892	1.43191	1.58988	1.51224	0.97236	0.16222	1.32382	***********
09	1.46089	2.15011	0.73574	1.53483	1.36166	1.25558	1.26670	1.15256
10	1.17680	1.90141	1.36423	1.36410	1.16163	0.86211	1.59719	0.73593
11	1.82397	1.78020	1.60078	0.48211	1.03656	1.58892	2.22493	1.57955
12	1.06991	0.61020	2.16980	1.81498	1.18553	1.33350	1.62424	1.48845
13	0.42425	1.33924	1.32111	1.46676	0.86154	0.35953	0.98242	0.37649
14	1.18979	2.23468	2.17511	2.26996	1.83550	1.00496	1.85454	0.88633
15	1.62072	1.97661	1.76510	1.09120	1.24370	1.30050	2.20153	1.23133

KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE

MATRIX OF (DIS)SIMILARITIES

	09	10	11	12	13	14	15
01	1.46089	1.17680	1.82397	1.06991	0.42425	1.18979	1.62072
02	2.15011	1.90141	1.78020	0.61020	1.33924	2.23468	1.97661
03	0.73574	1.36423	1.60078	2.16980	1.32111	2.17511	1.76510
04	1.53483	1.36410	0.48211	1.81498	1.46676	2.26996	1.09120
05	1.36166	1.16163	1.03656	1.18553	0.86154	1.83550	1.24370
06	1.25558	0.86211	1.58892	1.33350	0.35953	1.00496	1.30050
07	1.26670	1.59719	2.22493	1.62424	0.98242	1.85454	2.20153
08	1.15256	0.73593	1.57955	1.48845	0.37649	0.88633	1.23133
09	***********	0.71220	1.38690	2.22802	1.04247	1.56389	1.22404
10	0.71220	***********	1.20345	2.08123	0.87373	1.09058	0.71205
11	1.38690	1.20345	***********	2.21082	1.58997	2.20676	0.78349
12	2.22802	2.08123	2.21082	***********	1.31314	2.24171	2.33871
13	1.04247	0.87373	1.58997	1.31314	***********	1.17875	1.40449
14	1.56389	1.09058	2.20676	2.24171	1.17875	***********	1.58679
15	1.22404	0.71205	0.78349	2.33871	1.40449	1.58679	***********

1130-PROGRAM BY J.G. BLOM AND L.W.A. VAN HERPT

KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE

MATRIX OF DISTANCES

	01	02	03	04	05	06	07	08
01	0.00000	1.15993	1.70628	1.63785	0.95010	0.33306	1.19839	0.47146
02	1.15993	0.00000	2.13878	1.34485	0.83560	1.27741	1.89904	1.43710
03	1.70628	2.13878	0.00000	1.61795	1.46507	1.63753	1.08440	1.58335
04	1.63785	1.34485	1.61795	0.00000	0.72659	1.47746	2.09315	1.51074
05	0.95010	0.83560	1.46507	0.72659	0.00000	0.87387	1.54688	0.95986
06	0.33306	1.27741	1.63753	1.47746	0.87387	0.00000	1.32630	0.17374
07	1.19839	1.89904	1.08440	2.09315	1.54688	1.32630	0.00000	1.32100
08	0.47146	1.43710	1.58335	1.51074	0.95986	0.17374	1.32100	0.00000
09	1.46506	2.13878	0.73052	1.52251	1.36358	1.27595	1.27595	1.15993
10	1.17360	1.89905	1.36553	1.36553	1.15992	0.87094	1.59368	0.73052
11	1.81562	1.76392	1.61299	0.48334	1.04228	1.58335	2.23109	1.56485
12	1.08426	0.60682	2.15394	1.81038	1.17361	1.33688	1.62866	1.50066
13	0.41457	1.33688	1.31875	1.47026	0.85570	0.36344	0.98223	0.37459
14	1.19458	2.25664	2.15760	2.27696	1.83041	1.01407	1.83041	0.89050
15	1.62839	2.01062	1.76392	1.13636	1.27595	1.31047	2.19663	1.22467

KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE

MATRIX OF DISTANCES

	09	10	11	12	13	14	15
01	1.46506	1.17360	1.81562	1.08426	0.41457	1.19458	1.62839
02	2.13878	1.89905	1.76392	0.60682	1.33688	2.25664	2.01062
03	0.73052	1.36553	1.61299	2.15394	1.31875	2.15760	1.76392
04	1.52251	1.36553	0.48334	1.81038	1.47026	2.27696	1.13636
05	1.36358	1.15992	1.04228	1.17361	0.85570	1.83041	1.27595
06	1.27595	0.87094	1.58335	1.33688	0.36344	1.01407	1.31047
07	1.27595	[illegible]	2.23109	1.62866	0.98223	1.83041	2.19663
08	1.15993	[illegible]	1.56485	1.50066	0.37459	0.89050	1.22467
09	0.00000	0.71783	1.37356	2.23109	1.05518	1.56485	1.19839
10	0.71783	0.00000	1.19458	2.08561	0.87387	1.08440	0.68853
11	1.37356	1.19458	0.00000	2.20310	1.58578	2.19663	0.80741
12	2.23109	2.08561	2.20310	0.00000	1.31875	2.26407	2.36813
13	1.05518	0.87387	1.58578	1.31875	0.00000	1.17360	1.40688
14	1.56485	1.08440	2.19663	2.26407	1.17360	0.00000	1.56485
15	1.19839	0.68853	0.80741	2.36813	1.40688	1.56485	0.00000

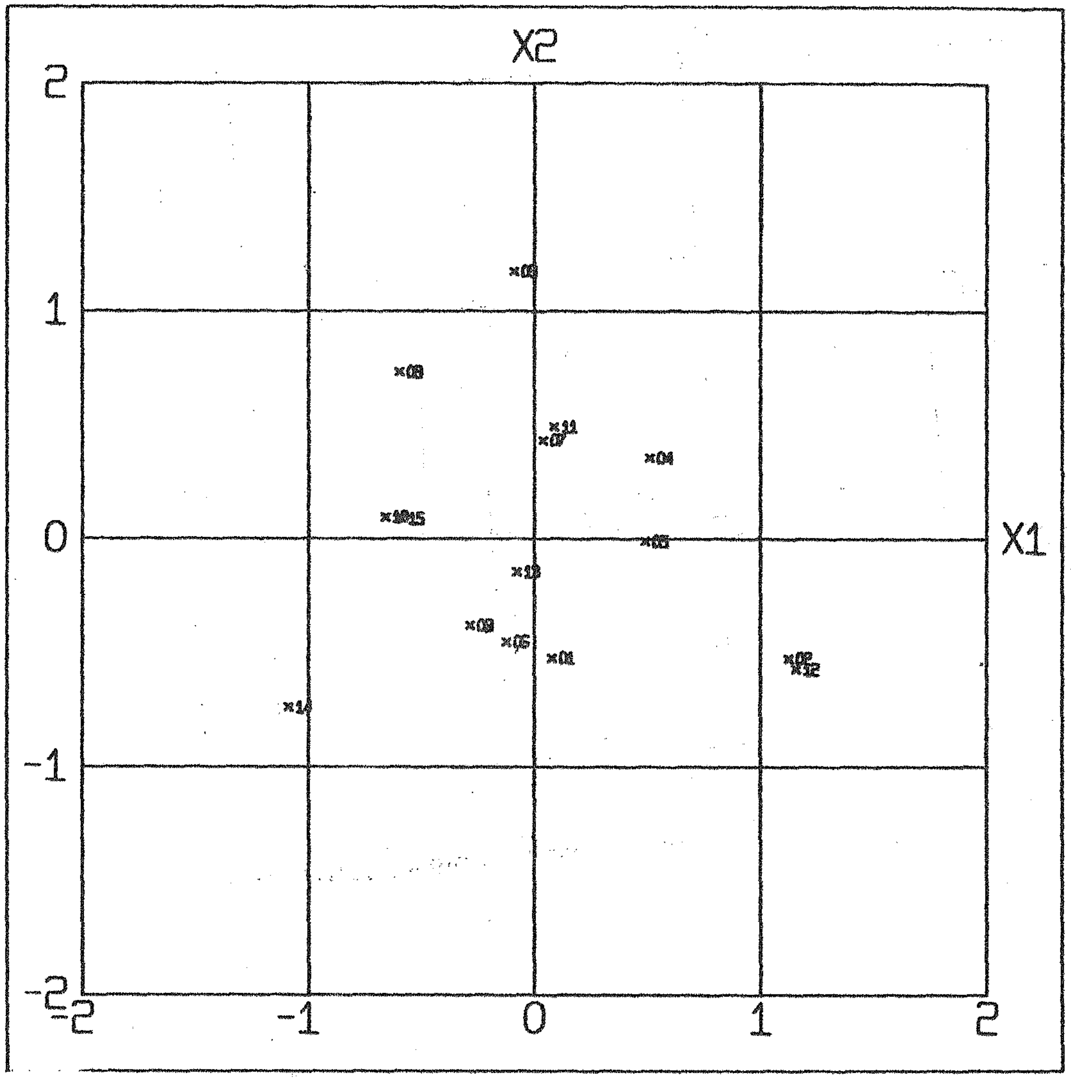
KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE

CONFIGURATION

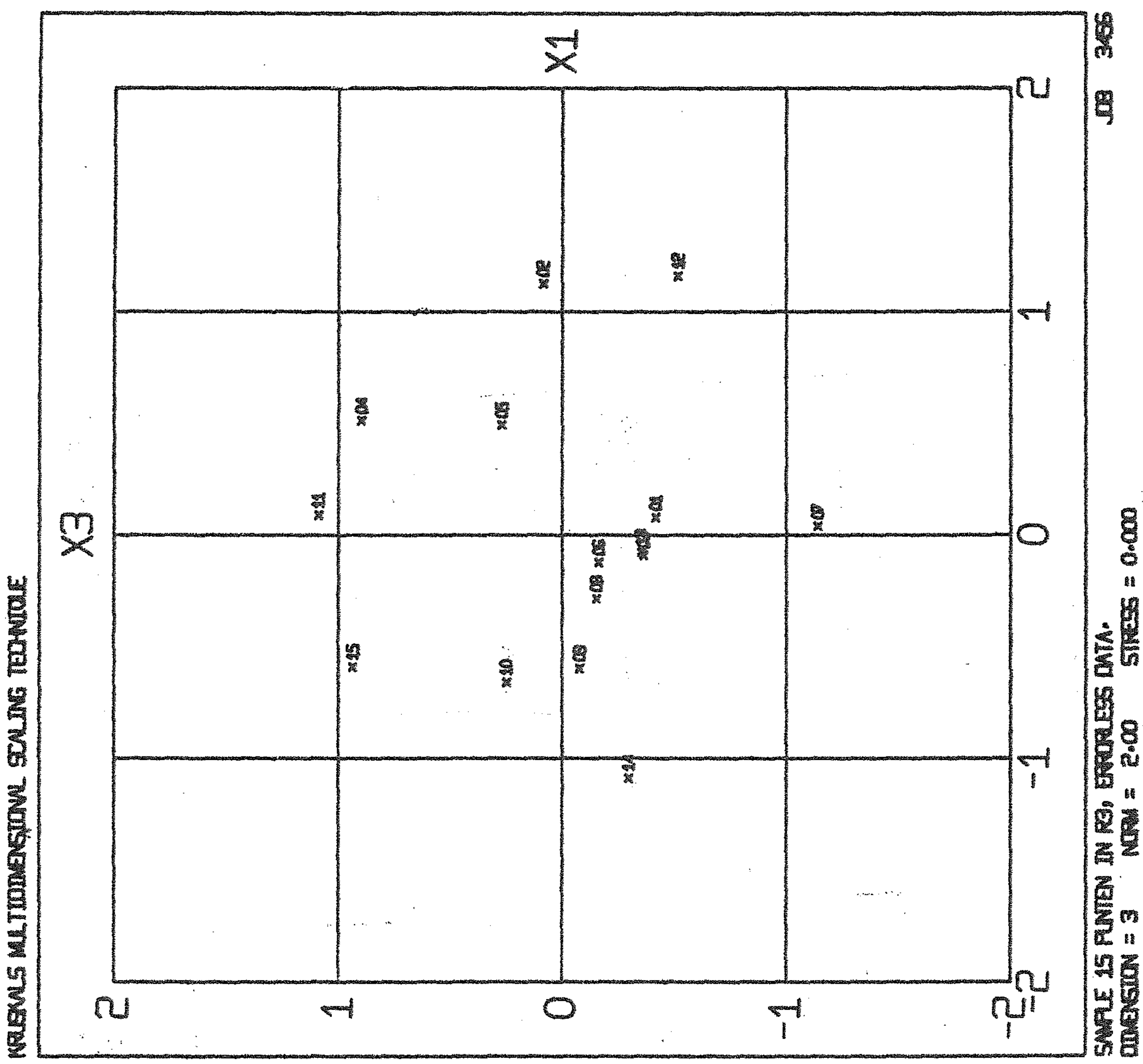
		1	2	3
1	01	0.08166	-0.52372	-0.41981
2	02	1.12620	-0.52792	0.08451
3	03	-0.08996	1.17302	-0.36502
4	04	0.51349	0.35458	0.89345
5	05	0.49408	-0.01274	0.26684
6	06	-0.12378	-0.45221	-0.16760
7	07	0.04041	0.43011	-1.14414
8	08	-0.28336	-0.38449	-0.15589
9	09	-0.59577	0.72995	-0.07951
10	10	-0.66075	0.09398	0.24698
11	11	0.09156	0.49134	1.08552
12	12	1.16014	-0.57420	-0.51958
13	13	-0.07637	-0.14563	-0.35695
14	14	-1.08727	-0.73916	-0.30062
15	15	-0.59028	0.08711	0.93186
VARIANCE		6.17302	5.24642	3.58053
PERCENT		41.15349	34.97617	23.87025

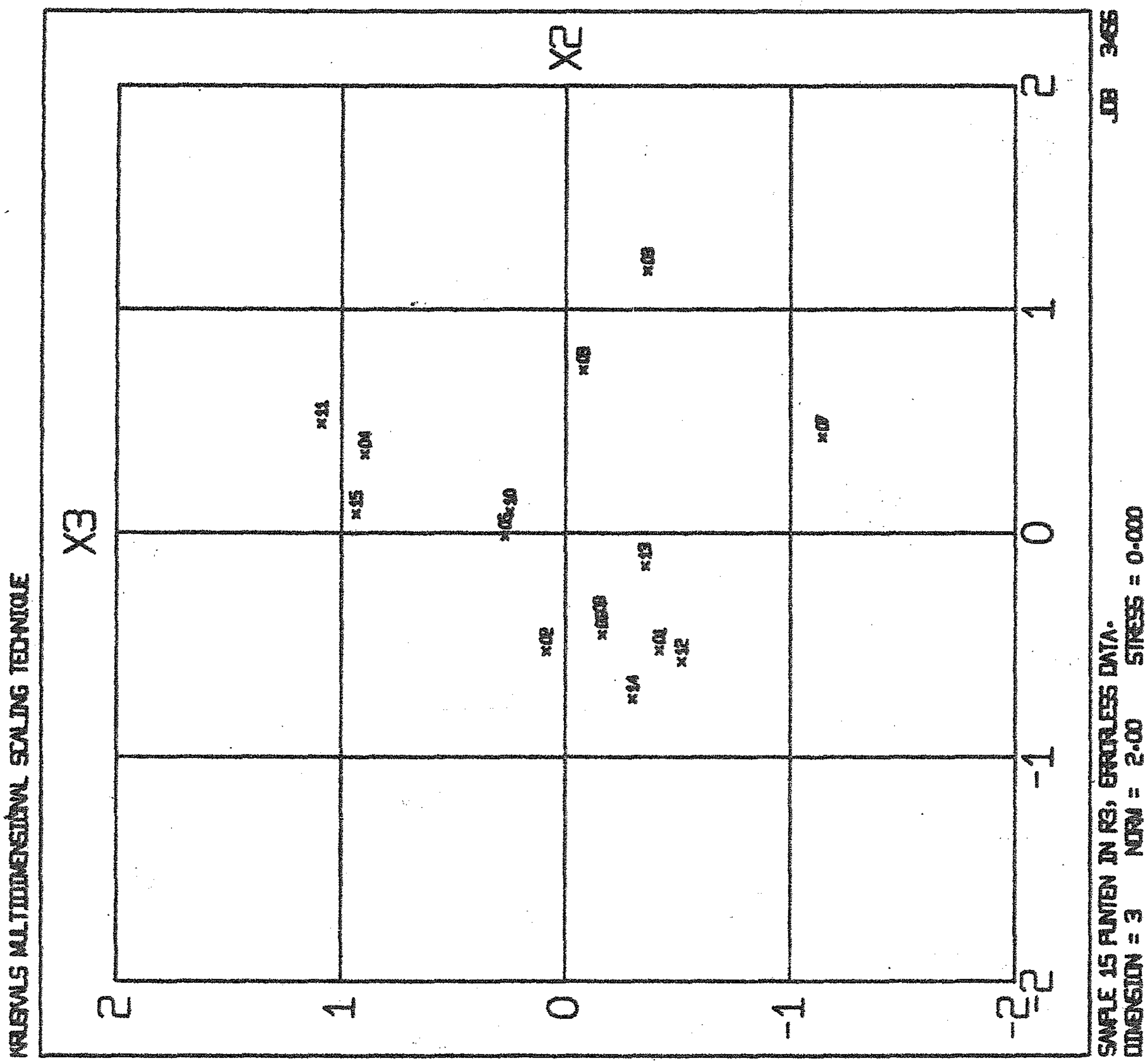
Page 10 containing
iteration information
about 'stress'
is omitted.

KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE



SAMPLE 15 PUNTEN IN R3, ERRORLESS DATA. JOB 3456
DIMENSION = 3 NORM = 2.00 STRESS = 0.000


KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE
X3
X1
SAMPLE 15 PUNTEN IN R3, ERRORLESS DATA.
DIMENSION = 3 NORM = 2.00 STRESS = 0.000
JOB 3456


KRUSKALS MULTIDIMENSIONAL SCALING TECHNIQUE
X3
X2
SAMPLE 15 PUNTEN IN R3, ERRORLESS DATA.
DIMENSION = 3 NORM = 2.00 STRESS = 0.000
JOB 3456

REFERENCES

Blom, J.G. and Herpt, L.W.A. van; Schaaltechnieken I -
Thurstone's Schaalmodel - Version 2, level 0 -
Programma voor IBM 1130
IFA-publication no. 41.2.

Blom, J.G. and Herpt, L.W.A. van; Schaaltechnieken I -
Thurstone's Schaalmodel - Version 2, level 0 -
Listings en Flowcharts
IFA-publication no. 42.2.

Blom, J.G. and Herpt, L.W.A. van; Schaaltechnieken II -
Schaaltechniek voor Categorische Data -
Version 1, level 0 -
Programma voor IBM 1130
IFA-publication no. 43.1.

Blom, J.G. and Herpt, L.W.A. van; Schaaltechnieken II -
Schaaltechniek voor Categorische Data -
Version 1, level 1 -
Listings and Flowcharts
IFA-publication no. 44.1.

Blom, J.G. and Herpt, L.W.A. van; Schaaltechnieken III -
Kruskals Multidimensionale Schaaltechniek -
Version 1, level 0 -
Programma voor IBM 1130
IFA-publication no. 45.0.

Blom, J.G. and Herpt, L.W.A. van; Schaaltechnieken III -
Kruskals Multidimensionale Schaaltechniek -
Version 1, level 0 -
Listings and Flowcharts
IFA-publication no. 47.0.

Diederich, G.W., Messick, S.J. and Tucker, L.R.; A General Least
Squares Solution for Succesive Intervals -
Psychometrika - Vol. 22, no. 2 - June 1957.

Kruskal, J.B. Multidimensional Scaling by Optimizing
Goodness of Fit to a Nonmetric Hypothesis
Psychometrika - Vol. 29, No. 1, March 1964.

Kruskal, J.B. Nonmetric Multidimensional Scaling: A numerical
method.
Psychometrika - Vol. 29, No. 2, June 1964.

Torgerson, W.S. Theory and Methods of Scaling -
New York: Wiley, 1967.